# Autonomous racecar control in head-to-head competition using Mixed-Integer QP

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# Introduction: the problem definition

- Ego Vehicle, EV and Leading Vehicle, LV perform the head-to-head competition for achieving fastest lap-time
- LV's trajectory is known by EV during its execution
- EV is responsible for the collision-avoidance during overtaking and its control strategy is calculated online

# Introduction: the base-solution

- In [Verschueren et al., 2016], a spatial Nonlinear Model Predictive Control (NMPC) method is proposed for time-optimal racing in a curvilinear coordinate system.

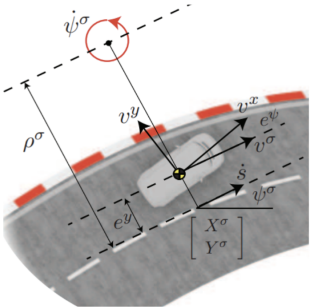


Figure: Curvilinear coordinate system. [Frasch et al., 2013]

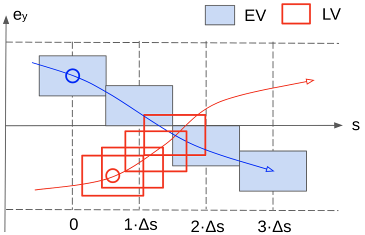



Figure: The prediction horizon in MPC.

# Introduction

## The NMPC problem formulation

$$
\begin{aligned}
&\min_{u_i(s)} \quad t_N \\
s.t. \quad & \xi_{i+1} = f_{\text{RK4}}^{\text{integration}}(\xi_i, u_i),\ i = 0, \dots, N-1 \\
& \xi_i \in [\underline{\xi}, \overline{\xi}],\ i = 1, \dots, N \\
& u_i \in [\underline{u}, \overline{u}],\ i = 0, \dots, N-1, \\
& (\text{collision-avoidance constraint})_i,\ i = 1, \dots, N, \\
& \text{where } \xi_i \text{ is the state vector,} \\
& \text{and } u_i \text{ is the control vector.}
\end{aligned} \tag{1}
$$

# Method

## Over-approximate vehicle's shape as a set

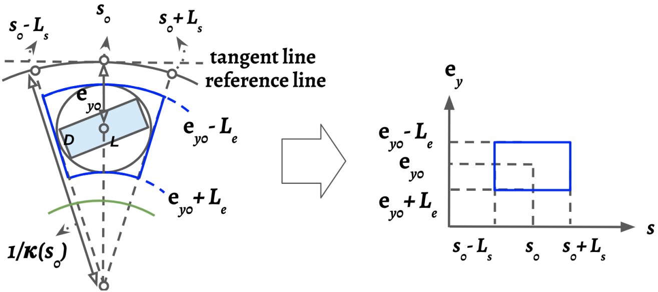



Figure: Vehicles' shape is firstly approximated as a circle and then projected as a set (blue sector) in the curvilinear coordinates system.

# Method

## Set up collision-avoidance constraint

- There should be no overlap between two shape approximation of EV and LV:

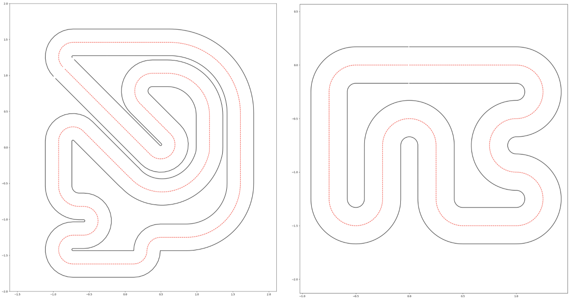
$$\begin{aligned} &(A)\ s_i^{LV} + (L_s)_i^{LV} \leq s_i^{EV} - (L_s)_i^{EV} \\ \vee\ &(B)\ s_i^{EV} + (L_s)_i^{EV} \leq s_i^{LV} - (L_s)_i^{LV} \\ \vee\ &(C)\ {e_y}_i^{LV} + (L_e)_i^{LV} \leq {e_y}_i^{EV} - (L_e)_i^{EV} \\ \vee\ &(D)\ {e_y}_i^{EV} + (L_e)_i^{EV} \leq {e_y}_i^{LV} - (L_e)_i^{LV} \end{aligned} \tag{2}$$

- Formulate it as a mixed-integer problem.

# Simulation results

## Experimental configuration

- use a 1:43 miniature racecar model (car size: $6.2cm \times 3cm$) and two different tracks (track width: $34cm$) [Liniger et al., 2015][Verschueren et al., 2016]
- suppose that EV and LV have the same dynamics and LV's optimal trajectory is pre-calculated



# Simulation results

| Track | Predeiction horizon length | # of collisions (in H2H*) | Mean lap time [s] | | Mean calc. time [ms/step] | |
|---|---|---|---|---|---|---|
| | | | H2H* | Single* | H2H* | Single* |
| 1 | 15 | 3/24 | **4.942** | 4.852 | **247** | 137 |
| | 30 | 0/24 | **4.899** | 4.773 | **905** | 245 |
| 2 | 15 | 0/45 | **10.278** | 10.189 | **244** | 118 |
| | 30 | 0/45 | **10.148** | 10.064 | **832** | 205 |

*H2H = Head-to-head mode, Single = Single car racing mode

- There might be collisions when horizon length is short.
- By increasing horizon length, we obtain better lap time while the calculation time increases too.

[animation]

# Conclusions and future works

- the effectiveness of the collision-avoidance time-optimal control algorithm.
- potential possibility to implement the algorithm with a short prediction horizon on a real-world vehicle or wheel-robot.
- potential possibility to reduce computation time for a long prediction horizon by simplifying the decision combinatorics etc.

Thanks for your listening!

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