

Introduction

We consider a dynamic device-to-device (D2D) communication model where transmitters and receivers adopt beamforming (BF). A continuous spatio-temporal model for the wireless network is analyzed, which combines a spatial point process and a dynamic birth-death process [1].

We model BF by using a uniform linear array (ULA) and derive an analytical stability condition of such a network. We show that the critical arrival rate increases with the number of antennas at the transmitter and the receiver.

Spatial Birth-Death Process

- D2D device pairs live in 2-D Euclidean square plane S .
- The appearing positions of the receivers are i.i.d. uniformly distributed in the plane S .
- The transmitter devices are uniformly distributed on the circles centered at the receivers of radius T .

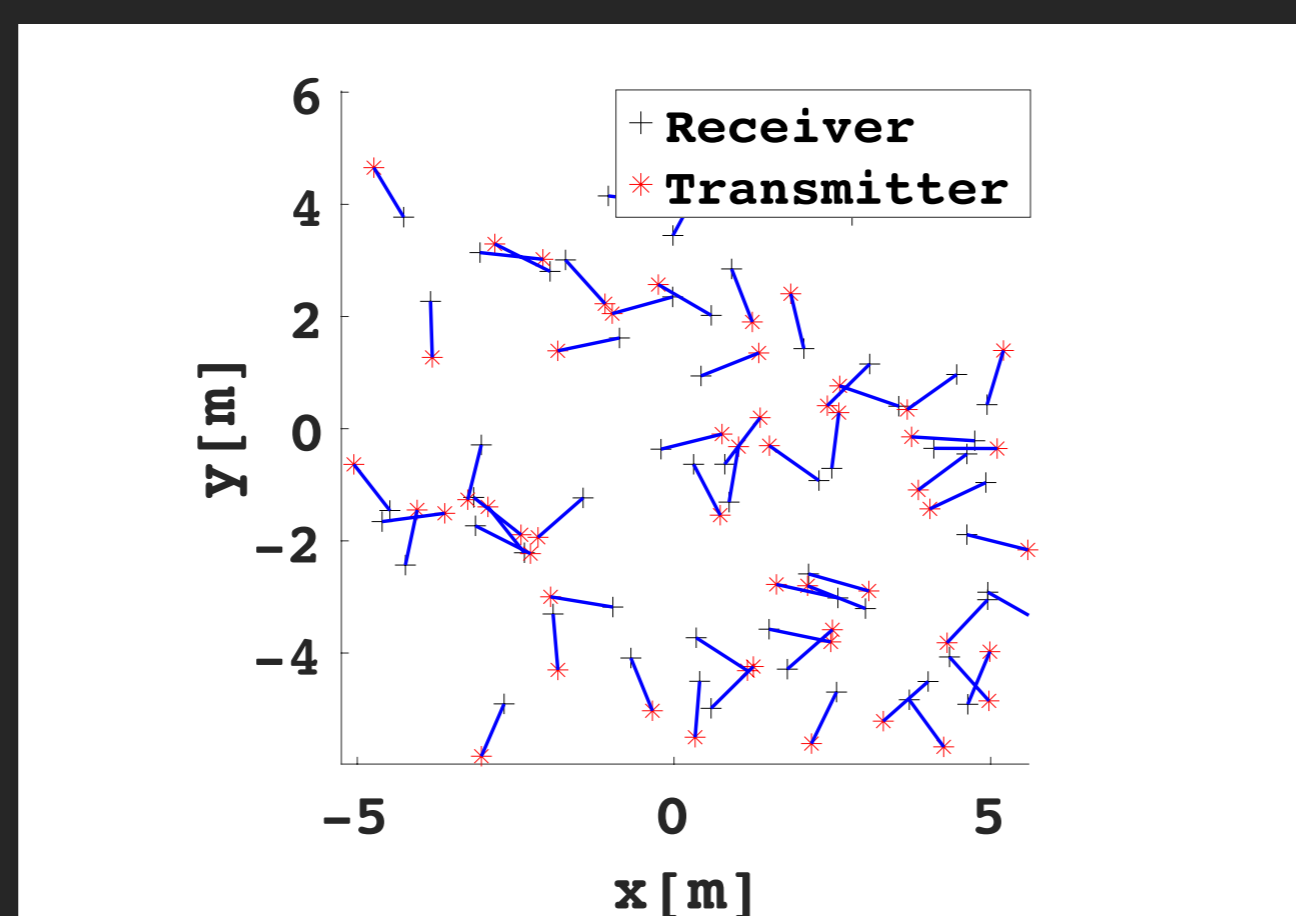
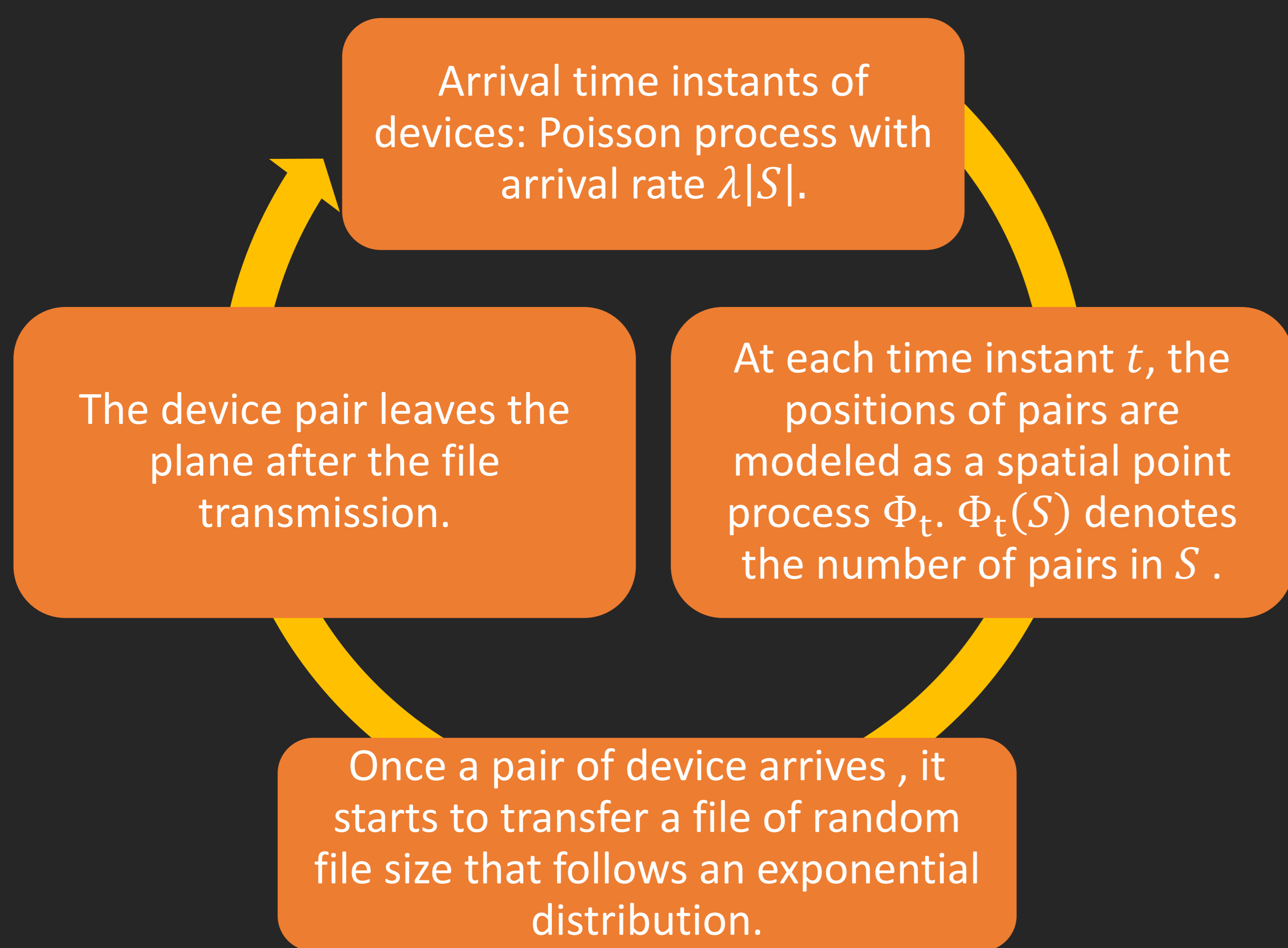


Figure 1. Distribution of device transmitter-receiver pairs.



Beamforming Model

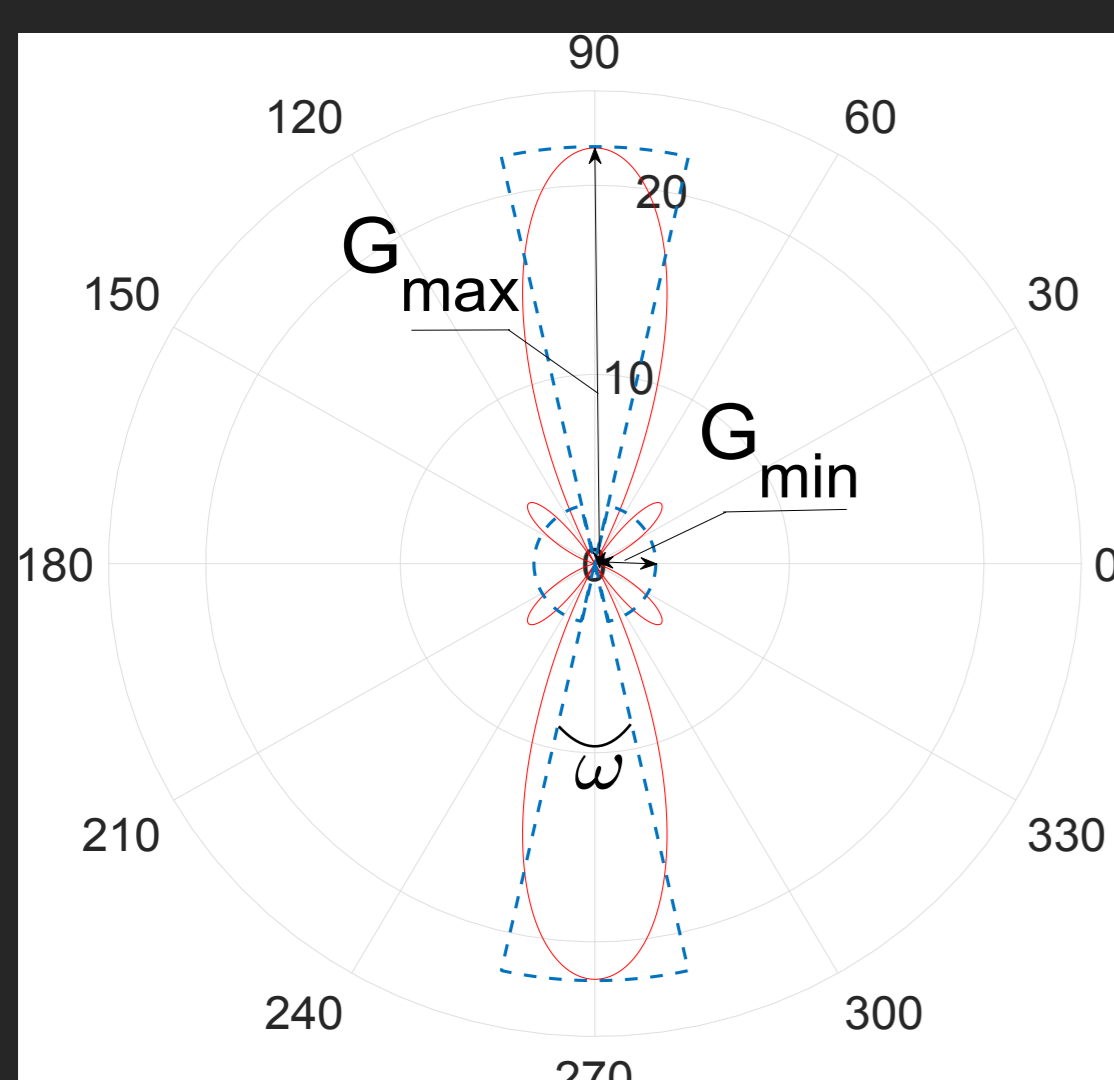


Figure 2. Uniform Linear Array beamforming (ULA BF) model (solid line) and approximated ULA BF model (dotted line).

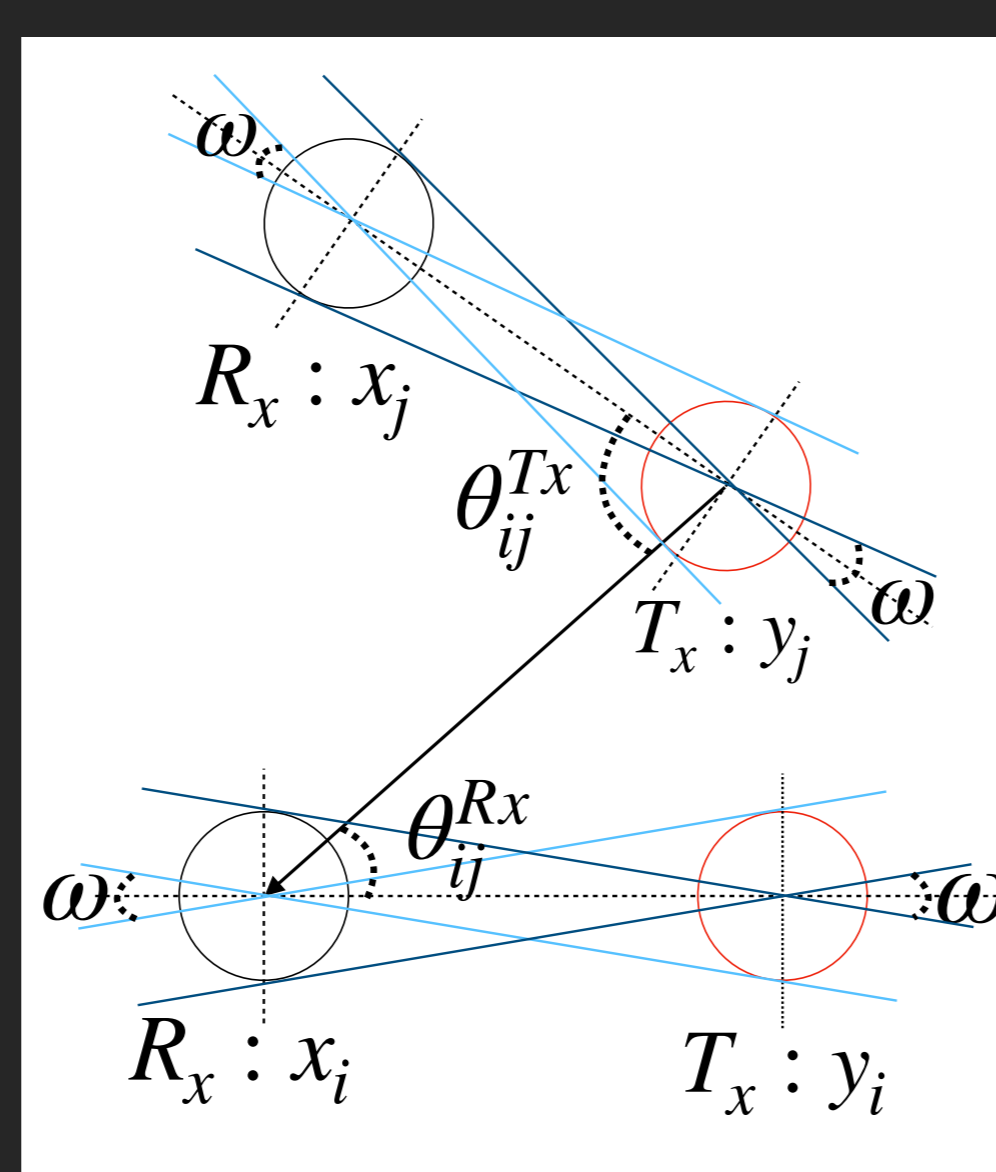


Figure 3. Schema of interference between pairs with BF. For each pair, boreside directions of the antenna arrays are aligned.

- G_{max} : Main beam of gain.
- G_{min} : Side beam of gain.
- n : Number of antenna elements.
- ω : Half power beam width (HPBW).

Antenna Types	Antenna gain	
	G_{max}	G_{min}
T_x	$2n$	ρ
R_x	$2n^2$	$n\rho$

Note T_x and R_x denote the transmitters and the receivers.

Critical Arrival Rate

Definition (Critical arrival rate) : The critical arrival rate λ_c is defined as the threshold of arrival rate such that, the spatial birth-death process Φ_t is stable if and only if $\lambda < \lambda_c$.

we define the critical arrival rate under the BF paradigm expressed as follows :

$$\lambda_c^{BF}(n) = \frac{4n^3 B l(T)}{\ln(2)La} \mathbb{E}[G(n)]$$

Where B denotes the bandwidth, L denotes the mean file size, $l(\cdot)$ denotes the path-gain function, and $a = \int_S l(x)dx$. Let $p = \frac{\omega}{\pi}$, the mean of antenna power gain is:

$$\mathbb{E}[G(n)] = 4n^3 p^2 + 4n^2 \rho p(1-p) + n\rho^2(1-p)^2$$

Theorem (Stability region) : Assuming the approximated ULA BF model with n antenna elements, if $\lambda > \lambda_c^{BF}(n)$, the spatial birth-death process Φ_t admits no stationary regime.

Simulation Results

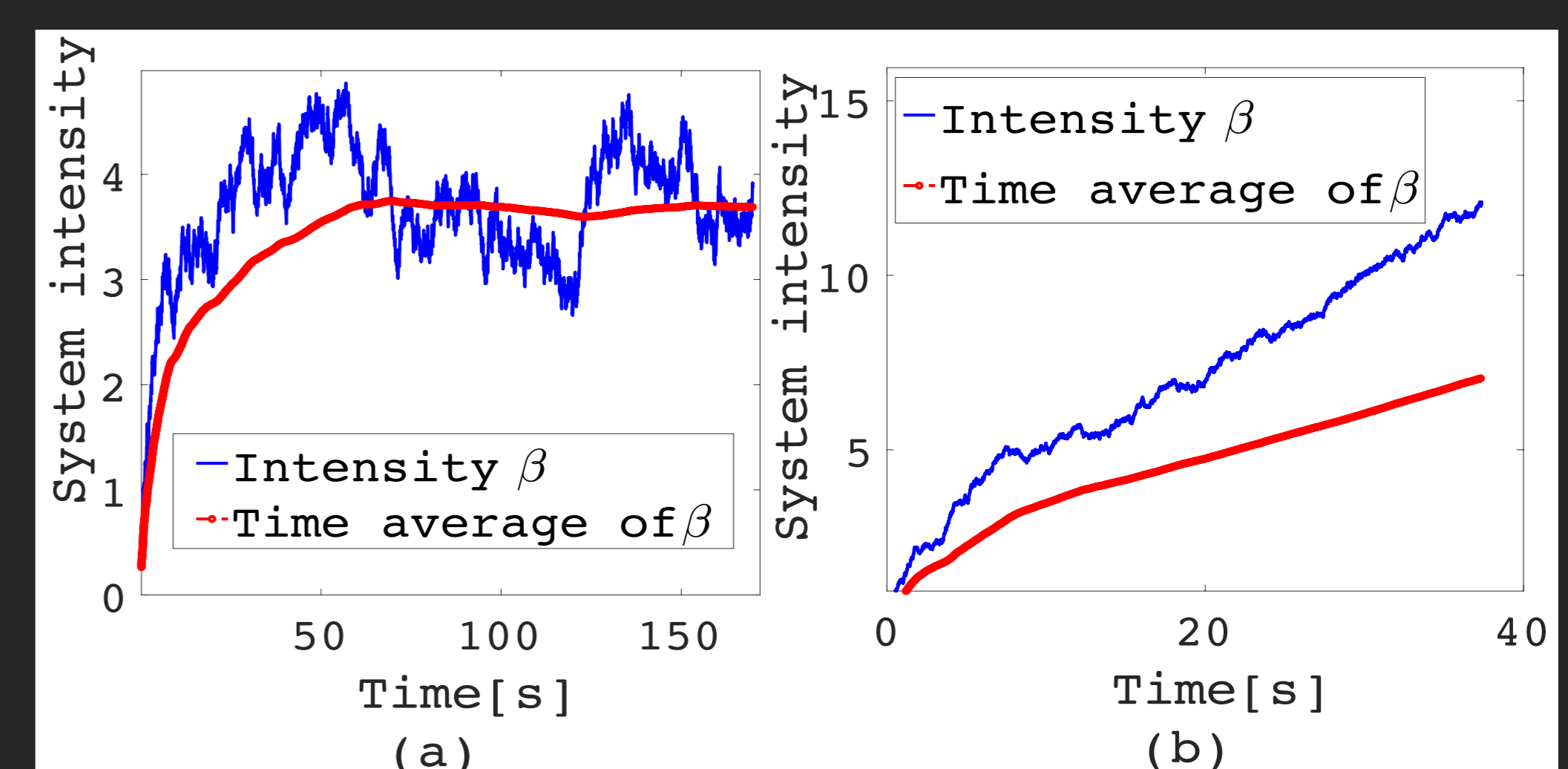


Figure 4. Intensity $\beta_t = \Phi_t(S)/|S|$, (a). $\lambda < \lambda_c^{BF}$, (b). $\lambda > \lambda_c^{BF}$.

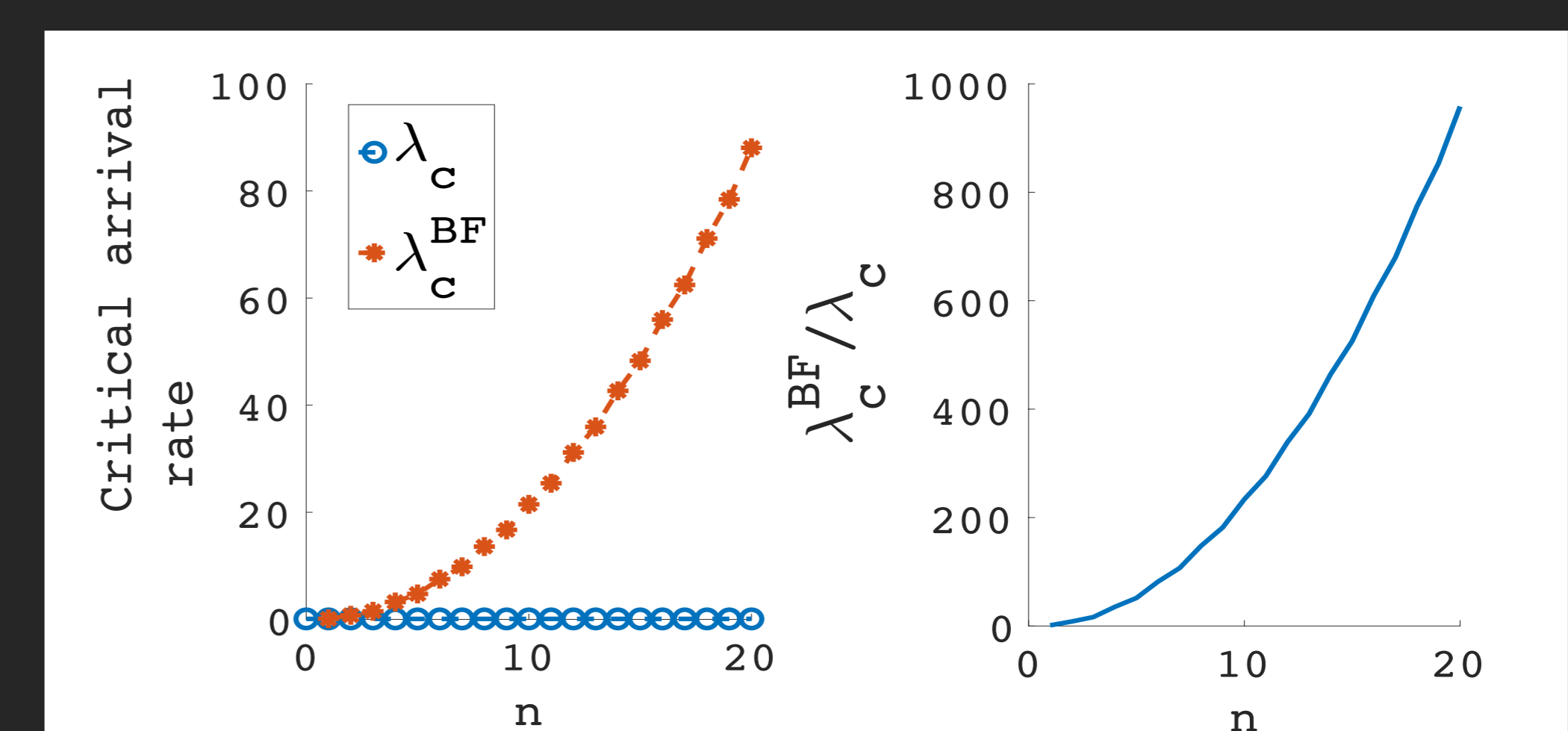


Figure 5. Critical arrival rate λ_c without BF and λ_c^{BF} with BF.

Conclusion

We present an extension of the spatial birth-death wireless network model proposed by Sankararaman and Baccelli to the scenario with BF, and provide an explicit expression of the critical arrival rate as a function of the number of antenna elements. Theoretical and numerical results show that BF expands significantly the stability region.

Reference

- [1] Abishek Sankararaman and Francois Baccelli, "Spatial birth-death wireless networks," *IEEE Transactions on Information Theory*, (6):3964–3982, June 2017.