New Methods for Tight Analysis of Population-based Evolutionary Algorithms

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Evolutionary Algorithms (EAs)

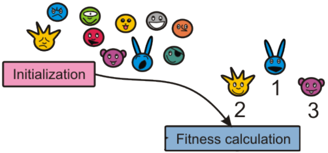
EAs are random search heuristics which are based on the concepts of the natural evolution:

- Mutation
- Crossover
- Selection
- Populations

Generic EA Scheme

Image source (downloaded in 2013): http://physiol.gu.se/maberg/images.html

Generic EA Scheme



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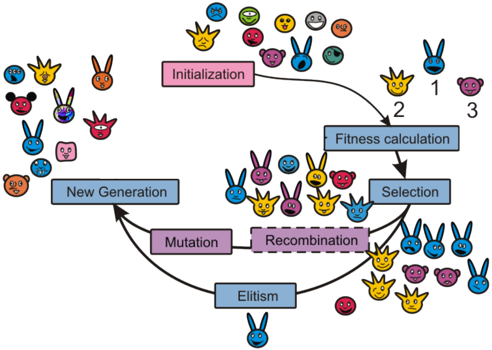

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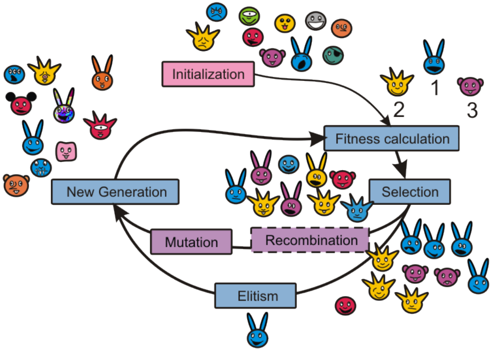

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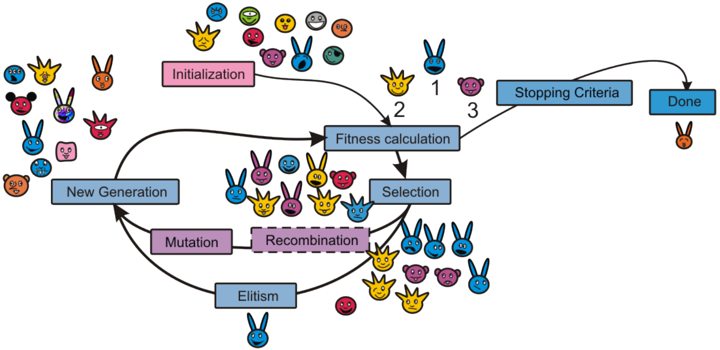

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Theory and Practice

Practice:

- Can solve hard problems with EAs
- Needs some advice on how to tailor an algorithm for a problem

Theory:

- Cannot analyse complicated problems
- Can give some valuable advice based on the analysis of easy problems

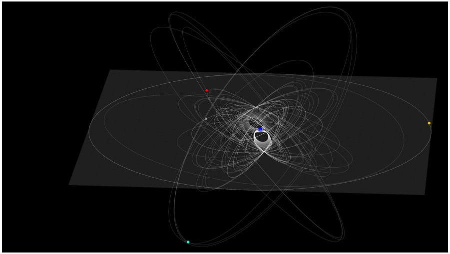
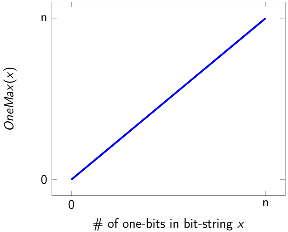


Illustration of the winner of GTOC 8 contest (image source: https://www.esa.int)



Goals of Theoretical Studies

- Understand working principles of EAs
- Improve existing EAs
- Propose new effective EAs

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The main tool of the theory is the runtime analysis via mathematical means

The Focus of the Thesis

We aim at the better understanding of the population-based EAs

- Complicated stochastic processes behind them
  - Lack of tools for their analysis
- Only few theoretical results existed
- Subject of great interest for practitioners

Contribution of the Thesis

New analysis methods

- The new method of the complete trees
- The new method for the analysis of no-drift processes
- Method for the precise analysis on plateaus
- New additive drift theorem with tail bounds

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New theoretical results

- Recommendations on how to set up parameters of EAs

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Parameters and Performance

Rally cars have multiple parameters which can be adjusted

- Breaks balance
- Transmission speed ranges
- Tires
- ...

Picture source:
https://toyotagazooracing.com/wrc/

Question: how to set the parameters for the best performance?

Recomendations from the Thesis Results

Static parameters choices

- Population size for the $(\mu + \lambda)$ EA: $\mu = O(\log(n))$ and $\lambda = O(\mu)$
- Population size for the (μ, λ) EA: $\mu \approx e\lambda$ and $\mu = \Omega(n^{3/4})$.
- When traversing plateaus of radius k: mutation rate should be $\frac{k}{en}$
- For the $(1 + (\lambda, \lambda))$ GA on JUMP_k: non-standard parameter setting $p = c = \sqrt{\frac{k}{n}}$ and $\lambda = \sqrt{\frac{n}{k}}^k$.

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Dynamic parameters choices

- We can effectively choose parameters randomly from a specific distribution (with proper scaling)
- After the thesis: we can do it with multiple parameters simultaneously

Summary

- In the thesis we proposed several new analysis methods
- With them we obtained some recommendations for the practical use of EAs
- We also proposed a new algorithm with the dynamic parameters choices and showed its efficiency on multiple problems

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Thank you!