Autonomous racecar control in head-to-head competition using Mixed-Integer QP

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Introduction: the problem definition

- Ego Vehicle, EV and Leading Vehicle, LV perform the head-to-head competition for achieving fastest lap-time
- LV's trajectory is known by EV during its execution
- EV is responsible for the collision-avoidance during overtaking and its control strategy is calculated online

Introduction: the base-solution

- In [Verschueren et al., 2016], a spatial Nonlinear Model Predictive Control (NMPC) method is proposed for time-optimal racing in a curvilinear coordinate system.

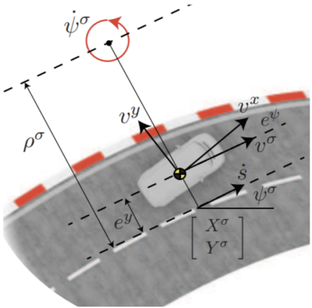


Figure: Curvilinear coordinate system. [Frasch et al., 2013]

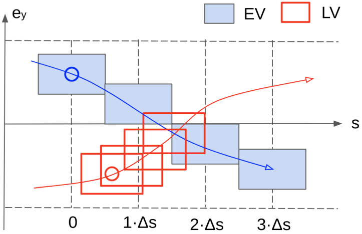


Figure: The prediction horizon in MPC.

Introduction

The NMPC problem formulation

$$
\begin{aligned}
&\min_{u_i(s)} \quad t_N \\
s.t. \quad \xi_{i+1} &= f^{\text{integration}}_{\text{RK4}}(\xi_i, u_i),\ i = 0, \ldots, N-1 \\
\xi_i &\in [\underline{\xi}, \overline{\xi}],\ i = 1, \ldots, N \\
u_i &\in [\underline{u}, \overline{u}],\ i = 0, \ldots, N-1, \\
(\text{collision-avoidance constraint})_i&,\ i = 1, \ldots, N, \\
\text{where } \xi_i &\text{ is the state vector,} \\
\text{and } u_i &\text{ is the control vector.}
\end{aligned} \tag{1}
$$

Method

Over-approximate vehicle's shape as a set

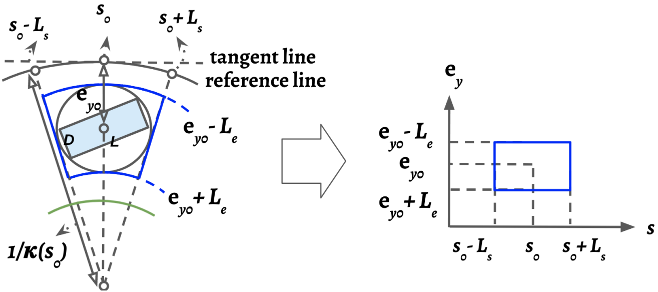


Figure: Vehicles' shape is firstly approximated as a circle and then projected as a set (blue sector) in the curvilinear coordinates system.

Method

Set up collision-avoidance constraint

- There should be no overlap between two shape approximation of EV and LV:

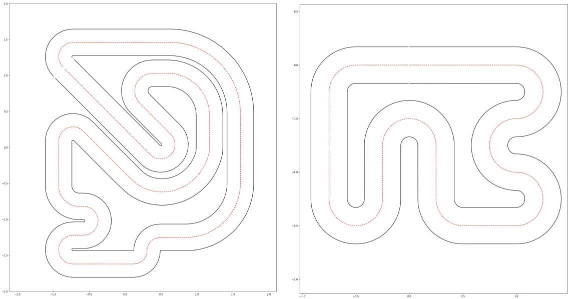
$$
\begin{aligned}
& (A)\ s_i^{LV} + (L_s)_i^{LV} \quad s_i^{EV} \quad (L_s)_i^{EV} \\
\_\ & (B)\ s_i^{EV} + (L_s)_i^{EV} \quad s_i^{LV} \quad (L_s)_i^{LV} \\
\_\ & (C)\ {e_y}_i^{LV} + (L_e)_i^{LV} \quad {e_y}_i^{EV} \quad (L_e)_i^{EV} \\
\_\ & (D)\ {e_y}_i^{EV} + (L_e)_i^{EV} \quad {e_y}_i^{LV} \quad (L_e)_i^{LV}
\end{aligned}
\qquad (2)
$$

- Formulate it as a mixed-integer problem.

Simulation results

Experimental configuration

- use a 1:43 miniature racecar model (car size: 6.2cm 3cm) and two different tracks (track width: 34cm) [Liniger et al., 2015][Verschueren et al., 2016]
- suppose that EV and LV have the same dynamics and LV's optimal trajectory is pre-calculated



Simulation results

Track	Predeiction horizon length	# of collisions (in H2H)	Mean lap time [s]		Mean calc. time [ms/step]	
			H2H	Single	H2H	Single
1	15	3/24	**4.942**	4.852	**247**	137
	30	0/24	**4.899**	4.773	**905**	245
2	15	0/45	**10.278**	10.189	**244**	118
	30	0/45	**10.148**	10.064	**832**	205

H2H = Head-to-head mode, Single = Single car racing mode

- There might be collisions when horizon length is short.
- By increasing horizon length, we obtain better lap time while the calculation time increases too.

[animation]

Conclusions and future works

- the effectiveness of the collision-avoidance time-optimal control algorithm.
- potential possibility to implement the algorithm with a short prediction horizon on a real-world vehicle or wheel-robot.
- potential possibility to reduce computation time for a long prediction horizon by simplifying the decision combinatorics etc.

Thanks for your listening!

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