Autonomous Racecar Control in Head-to-Head Competition using Mixed-Integer Quadratic Programming

Nan Li [1], Eric Goubault [2], Laurent Pautet [1], Sylvie Putot [2]

[1]LTCI, Télécom Paris, Institut Polytechnique de Paris [2]LIX, CNRS, Ecole Polytechnique, Institut Polytechnique de Paris

Introduction

- **Background**: This work deals with the control of an autonomous racecar (Ego Vehicle, EV) that should perform the fastest lap time on a track, while in presence of an opponent vehicle (Leading Vehicle, LV).
- **Method**: We propose a Nonlinear Model Predictive Control (NMPC) model under a minimum time objective, which integrates the opponent's trajectory as a collision-avoidance constraint.

System Dynamics and Curvilinear Coordinate

The reference line (track's center line) is parameterized with length s as a curviliner coordinate, in which:

- as in Fig. 1, we define: the linear and angular velocity - v_x, v_y, and ω; the position and orientation as the deviation from the reference line - e_y and e_ψ
- a dynamic bicycle model is established to capture the vehicle's dynamics (shown in Eq. (1)) in which d and δ are the control variables for motor and vehicle steering; $F_{F/R,x/y}$ are the front / rear side force along / vertical to tire; $\kappa(s)$ is the local curvature around projection point.
- we represent the time t as dependent variable: $\frac{d}{ds}t = \frac{1-e_y\cdot\kappa(s)}{v_x cos(e_\psi)-v_y sin(e_\psi)}$ and it is a direct objective for optimisation problem.
- the track constraint is a simple interval set, with fixed track width L, the relative position is simply constrained as: $e_y \in [-L, L]$.

$$\frac{d}{ds}\begin{bmatrix} e_y \\ e_\psi \\ v_x \\ v_y \\ \omega \\ t \\ s \\ d \\ \delta \end{bmatrix} = \frac{1}{\dot{s}}\begin{bmatrix} v_x \sin(e_\psi) + v_y \cos(e_\psi) \\ \omega - \dot{s}\cdot\kappa(s) \\ \frac{1}{m}(F_{R,x} - F_{F,y}\sin\delta + m v_y \omega) \\ \frac{1}{m}(F_{R,y} + F_{F,y}\cos\delta - m v_x \omega) \\ \frac{1}{I_z}(l_f F_{F,y}\cos\delta - l_r F_{R,y}) \\ 1 \\ \dot{s} \\ \Delta d \\ \Delta\delta \end{bmatrix} \quad (1)$$

where $\dot{s} = \frac{v_x cos(e_\psi)-v_y sin(e_\psi)}{1-e_y\cdot\kappa(s)}$.

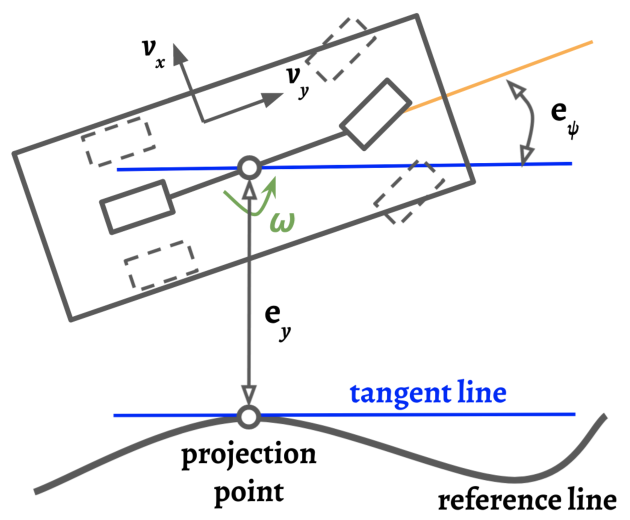

Figure 1. System dynamics.

Shape Approximation and Collision-Avoidance

As shown in Fig. 2, we define an over-approximation of the vehicle's occupied area and project it into curvilinear coordinate. The collision-avoidance constraint means no intersection between the occupied areas of EV and LV, i.e.

$$\begin{aligned}&[s_0^{EV} - L_s^{EV}, s_0^{EV} + L_s^{EV}] \times [e_{y0}^{EV} - L_e^{EV}, e_{y0}^{EV} + L_e^{EV}] \\ \cap&[s_0^{LV} - L_s^{LV}, s_0^{LV} + L_s^{LV}] \times [e_{y0}^{LV} - L_e^{LV}, e_{y0}^{LV} + L_e^{LV}] = \emptyset\end{aligned} \quad (2)$$

We define a mixed-integer form of the above constraints for EV at step i of the prediction horizon as following configurations: (A) EV is ahead of LV; (B) EV is behind LV; (C) EV is at the left of LV; (D) EV is at the right of LV, i.e. as written in Eq. (3).

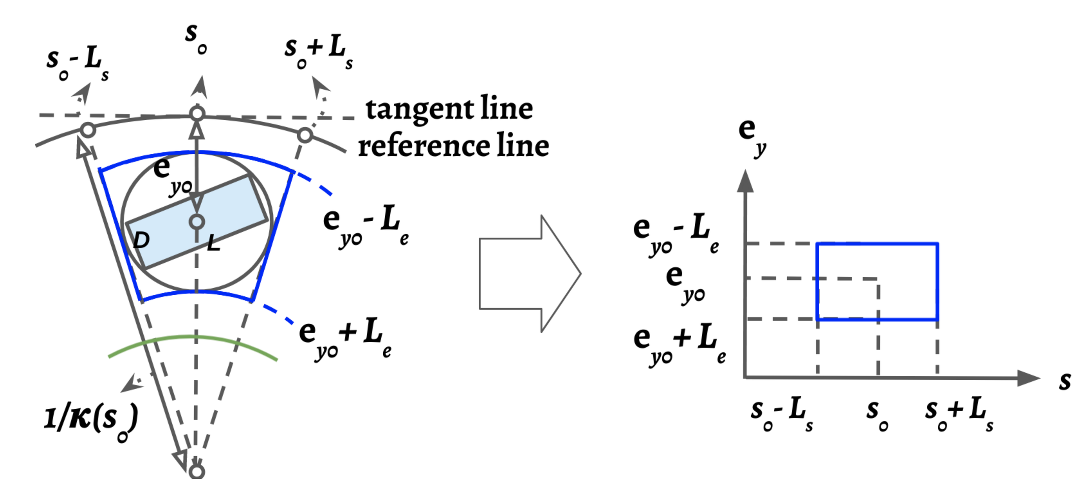

Figure 2. Curvilinear coordinate transformation

$$\begin{aligned}&(A)\ s_i^{LV} + (L_s)_i^{LV} \le s_i^{EV} - (L_s)_i^{EV} &&\vee\ (B)\ s_i^{EV} + (L_s)_i^{EV} \le s_i^{LV} - (L_s)_i^{LV} \\ \vee\ &(C)\ e_{yi}^{LV} + (L_e)_i^{LV} \le e_{yi}^{EV} - (L_e)_i^{EV} &&\vee\ (D)\ e_{yi}^{EV} + (L_e)_i^{EV} \le e_{yi}^{LV} - (L_e)_i^{LV}\end{aligned} \quad (3)$$

We refine them into 4 non-overlapping configurations, by adding to the cases (C) and (D) the condition that EV is neither totally ahead of LV nor totally behind of LV. Formally written as:

$$\begin{aligned}&(f_A(i) \le 0) \vee (f_B(i) \le 0) \\ \vee&(f_C(i) \le 0 \wedge (f_A(i) > 0 \wedge f_B(i) > 0)) \\ \vee&(f_D(i) \le 0 \wedge (f_A(i) > 0 \wedge f_B(i) > 0))\end{aligned} \quad (4)$$

Using big-M theory, we reduce the number of binary varibale from 4 to 2. In following equation, $a_1 = 1 + c_1 - c_2$, $a_2 = 1 - c_1 + c_2$, $a_3 = c_1 + c_2$, $a_4 = 2 - c_1 - c_2$, c_1 and c_2 are 2 binary variables.

$$\begin{cases} f_A(i) \le a_1 \cdot M \\ f_B(i) \le a_2 \cdot M \end{cases} \begin{cases} f_C(i) \le a_3 \cdot M \\ -f_A(i) \le a_3 \cdot M \\ -f_B(i) \le a_3 \cdot M \end{cases} \begin{cases} f_D(i) \le a_4 \cdot M \\ -f_A(i) \le a_4 \cdot M \\ -f_B(i) \le a_4 \cdot M \end{cases} \quad (5)$$

If $c_1 = 0, c_2 = 1$, the first constraint of Eq. (4) is active and other constraints are relaxed. If $c_1 = 1, c_2 = 0$, the second constraint is active. If $c_1 = c_2 = 0$, the third group of constraint is active. If $c_1 = c_2 = 1$, the last group of constraint is active.

Formulation of the Optimisation Problem

- Finding a control minimizing the lap time is expressed as an Optimal Control Problem (OCP).
- Piecewise constant control parameterization changes a continuous OCP into a Model Predictive Control (MPC) problem, which can be solved efficiently.
- We use a multiple shooting method for an horizon of N control-steps. The resulting sets of constraints can then be solved by Non-Linear Programming (NLP) optimisation.
- We solve this MPC problem by sequentially solving Quadratic Programs (QP) problem based on an exact Hessian matrix expansion [1].
- Combining with the constraint in Eq. (5), we formulate the optimisation problem as

$$\begin{aligned}\min_{u_i(s)}\ & t_N \\ s.t.\ \ & \xi_{i+1}' = f_{dyn}(\xi_i, u_i),\ i = 0, ..., N \\ & \xi_i \in [\underline{\xi}, \overline{\xi}],\ i = 0, ..., N+1 \\ & u_i \in [\underline{u}, \overline{u}],\ i = 0, ..., N,\end{aligned} \quad (6)$$

where ξ_i is the state vector $[e_y, e_\psi, v_x, v_y, \omega, t, s, d, \delta]$ and u_i is the control vector $[\Delta d, \Delta\delta]$.

Simulation result

In this work, we use the identification parameters of a 1:43 miniature racecar [2] with the maximum speed at $1.6m/s$. It is potentially possible to implement a similar algorithm with some modification on the F1Tenth racecar [3] which allows a maximum speed at $20m/s(\ 70km/h)$.

	Horizon length	# of cases where collision happens	Average lap time [s]	Average calculation time per step before overtaking [ms]
Track 1	15	3	4.942	247.1
	30	0	4.899	904.9
Track 2	15	0	10.278	243.5
	30	0	10.148	831.9

The above table summarizes the simulation result of head-to-head competition on 2 tracks, from which we observed the following features: a longer horizon yields better lap time but requires a higher computation cost; a shorter horizon has the risk of collision.

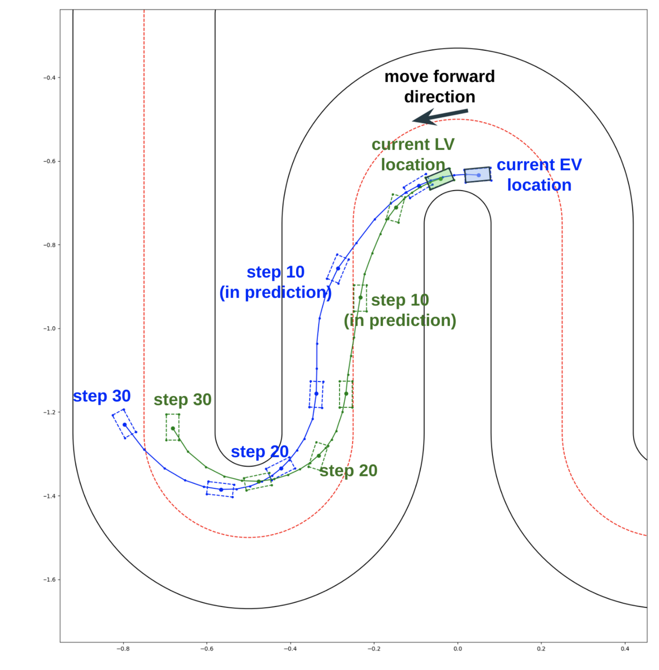

Figure 3. The trajectory of EV and LV in a typical scenario

A typical example of calculation result for a given progress point is presented in Fig. 3. We observed the following behavior in EV's prediction horizon. EV plans:

1. to follow LV from step 1 to 10 (2nd condition in Eq. (4) is active)
2. to overtake LV at the right from step 11 to 19 (4th condition in Eq. (4) is active)
3. to be completely ahead of LV at step 20 (1st condition in Eq. (4) is active)
4. to keep this advantage until step 26, to keep at the left of LV at the last 4 steps (3th condition in Eq. (4) is active).

Conclusion and Discussion

- The previous result demonstrated the effectiveness of the algorithm.
- However, with the current configuration, the average progress time per step is lower than the calculation time per step. It shows the difficulty of the implementation on a real-world racecar of the NMPC-based controller with the MIQP method encoding non-collision constraints. There are several possibilities to solve this problem in the future:
  - to simplify the decision combinatorics
  - to explore the problem structure of MIQP method and take the advantage of the multi-core system
- On another hand, low-speed car-like robot (such as two-wheel differential-drive service robot), which allows a relatively slow calculation time, could benefit from this proposed algorithm.

References

[1] Verschueren et al., 2016, time-optimal race car driving using an online exact hessian based nonlinear mpc algorithm.

[2] Liniger et al., 2015, optimization-based autonomous racing of 1: 43 scale rc cars.

[3] F1tenth racecar, `https://f1tenth.org`.