# Reachability Analysis with Measurable Time-Varying Uncertainties

François Bidet, Éric Goubault, Sylvie Putot

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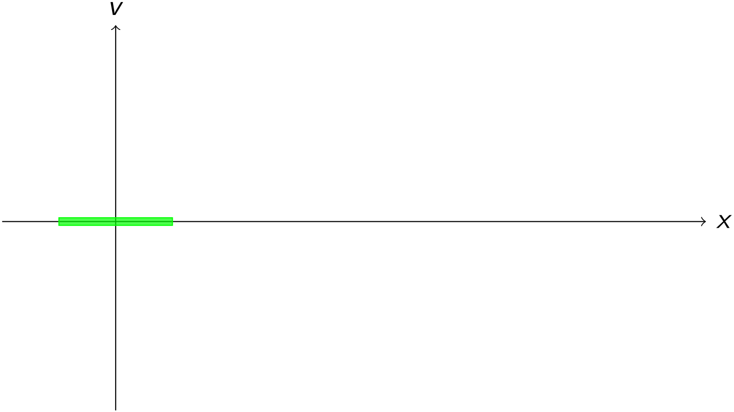

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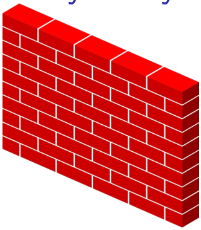

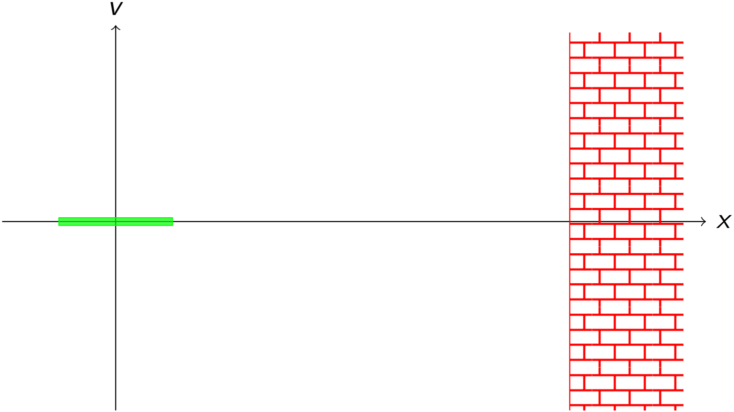

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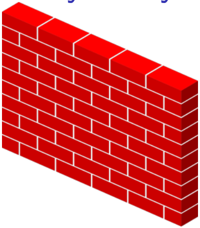

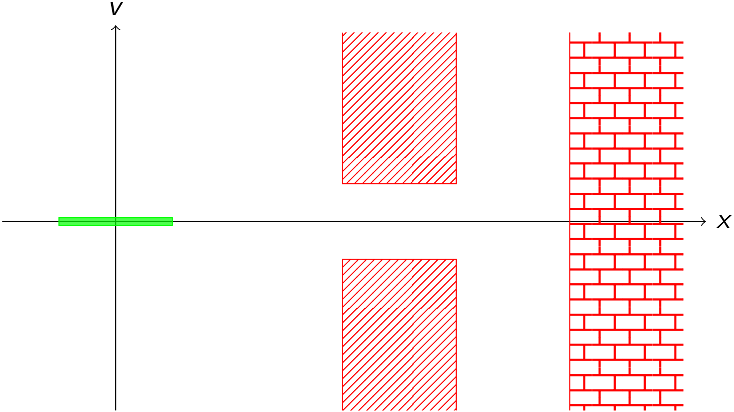

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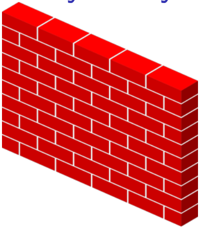

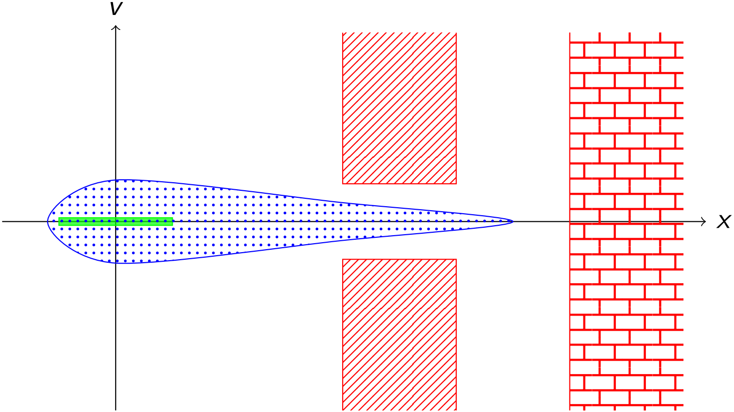

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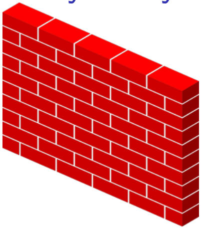

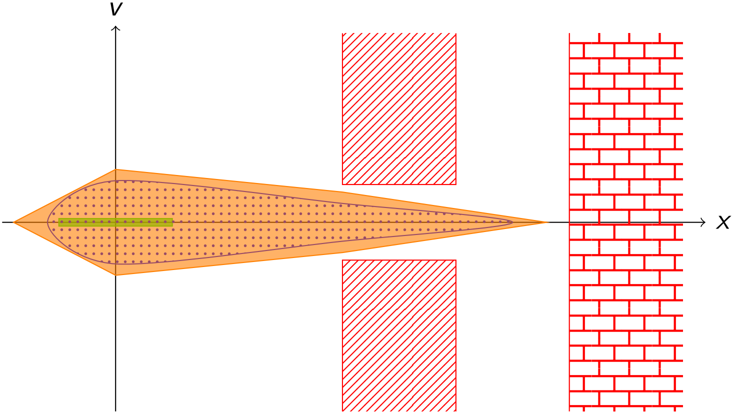

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## Bounded uncertainties

constant $\subset$ continuous $\subset$ Riemann-integrable

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$$\begin{cases} \dot{x}(t) = g(u(t)) \cdot h(t, x(t)) \\ x(0) \in \mathcal{X}_0 \\ u(t) \in \mathcal{U} \end{cases} \qquad \text{with } u \text{ Lebesgue-measurable}$$

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   $\rightarrow$ parametrized by $x(0)$, $A$ and $B$ (constant uncertainties)

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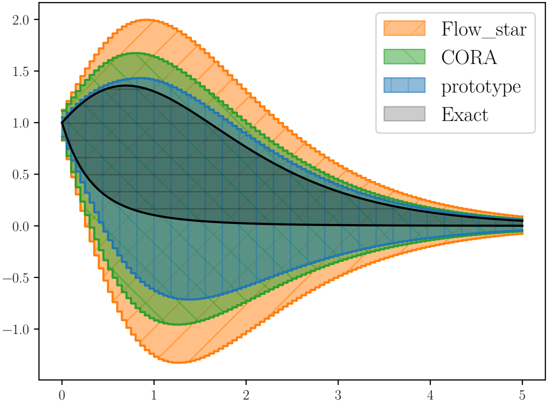
3. Compute over-approximation

# Example: Nonlinear

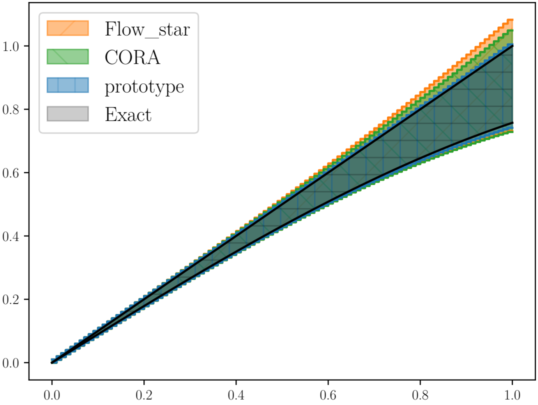
$$\begin{cases} \dot{x}(t) = -x(t) - x(t)y(t)u(t) \\ \dot{y}(t) = -y(t) \end{cases} \quad \text{with} \quad \begin{cases} x(0) = 1 \\ y(0) = 2 \\ u(t) \in [-1, 1] \end{cases}$$




Over-approximations of $x(t)$

# Example: Dubins car

$$\begin{cases} \dot{x}(t) = u_1(t)\cos(z(t)) \\ \dot{y}(t) = u_1(t)\sin(z(t)) \\ \dot{z}(t) = u_2(t) \end{cases} \quad \text{with} \quad \begin{cases} x(0) = y(0) = z(0) = 0 \\ u_1(t) \in [0.9, 1] \\ u_2(t) \in [0, 1] \end{cases}$$

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# Conclusion

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# Thank you for your attention