# Syntactic Regions for Deadlock Computation

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Many models were introduced to study the state space of programs in geometrical representations while keeping features of executions. In this context we translate models and algorithms based on topological spaces in a more natural setting : the syntax of programs, from the observation that positions in a program can be described as partial explorations of the program. These positions can be ordered as to define set of intervals such that under reasonable assumption they possess a cannonical form (which corresponds to covering a space by maximal intervals), and the structure needed to study it.

## A Skeleton Language for Concurrency

We introduce generic *programs* $P$ generated by the following grammar:

| | | | |
|---|---|---|---|
| $P, Q$ | $::=$ | $\alpha$ | Actual operations of our language: $x \leftarrow 1 \dots$ |
| | $\mid$ | $P\,;Q$ | Sequential composition : first $P$, then $Q$ |
| | $\mid$ | $P^*$ | Non-deterministic loop : **while** $\cdot$ **do** $P$ |
| | $\mid$ | $P{+}Q$ | Non-deterministic choice : **if** $\cdot$ **then** $P$ **else** $Q$ |
| | $\mid$ | $P \parallel Q$ | Parallel execution of two programs $P$ and $Q$ |

## Programs as Posets of Executions

A **position** in a program describes where we are during its execution. Programs have two states : unexplored $\bot$ and executed $\top$. Loops $P^*$ have positions define for each loop $P^n$, $n \in \mathbb{N}$. Furthermore

- In a sequence $P\,;Q$, $P$ must be explored fully before exploring $Q$
- For a branching, only one can be explored i.e. $p{+}q$ implies $p$ or $q = \bot$.

We order these positions according to execution reachability : $p \leq q$ if and only if $q$ corresponds to a position reachable by execution from the position $p$.

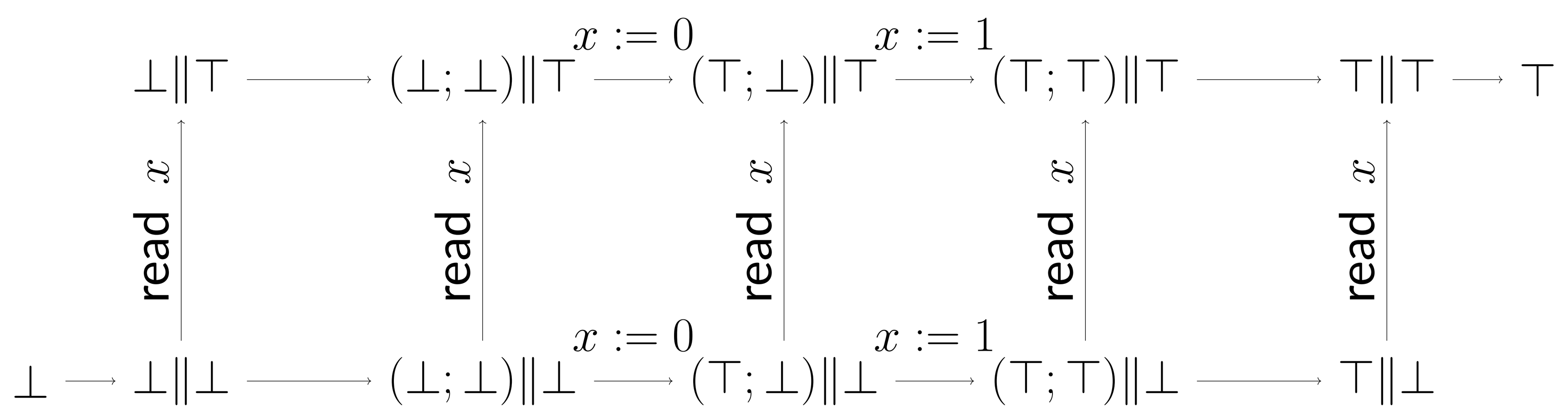



Figure 1. Set of Positions of ((x := 0) ; (x:=1)) ‖ (read x)

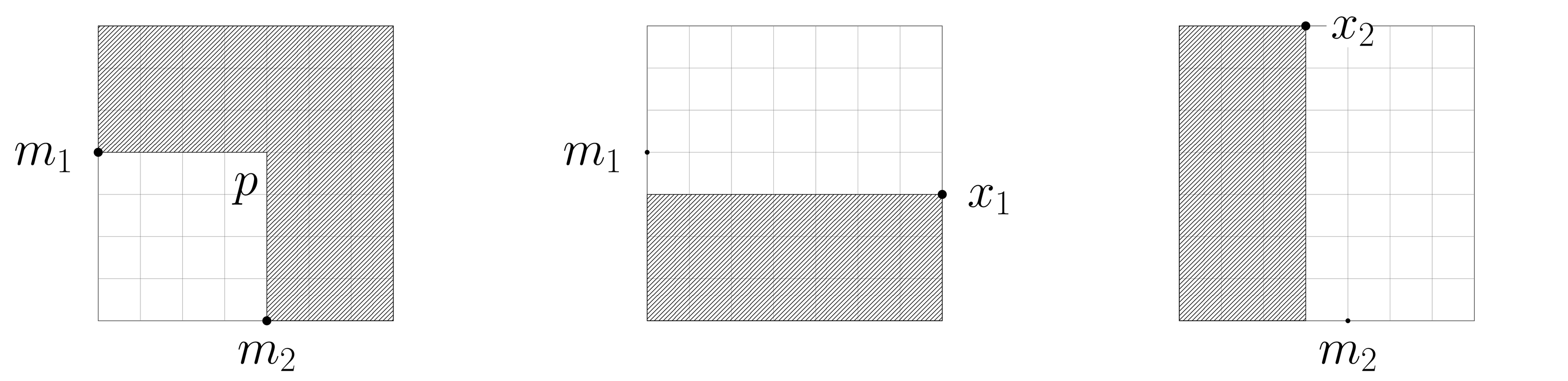
## Positions wrt Execution

- **Forbidden** : Syntactically correct but non-executable (concurrent writing...)
- **Authorized** : Any synctactically correct position that is not forbidden.
- **Reachable** : Authorized + Can be reached during execution.
- **Deadlocks** : Reachable + No direct reachable successor.

## Properties of the Set of Positions $X$

- $(X, \leq)$ is lattice and well ordering (finite antichains and well-foundedness)
- For all elements $p$ of $X$, for all $m \in \min\{x \mid x \not\leq p\}$, the set of maximal elements of $\{x \mid x \not\geq m\}$ is finite and its downwards closure covers $\{x \mid x \not\geq m\}$.

These are the necesary properties of a poset for all following algorithms.




$\min\{x \mid x \not\leq p\} = \{m_1, m_2\}$ $\quad\{x \mid x \not\geq m_1\} = \downarrow\{x_1\}$ $\quad\{x \mid x \not\geq m_2\} = \downarrow\{x_2\}$

Figure 2. $[\![0,6]\!] \times [\![0,6]\!]$ verifies the last property

## References

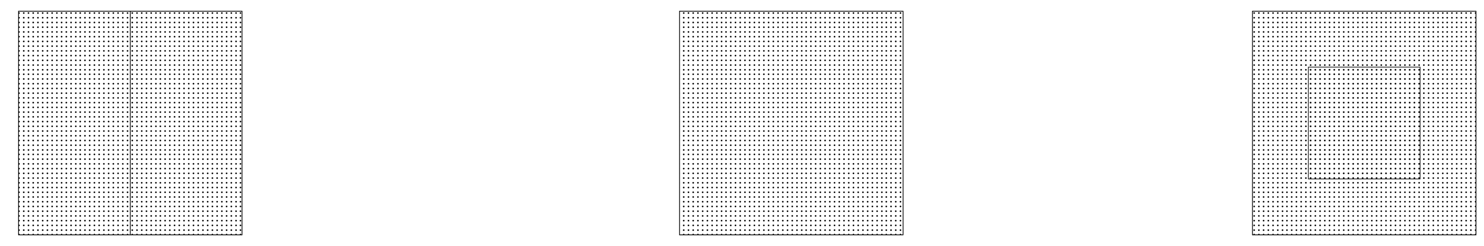
- Lisbeth Fajstrup et al. *Directed Algebraic Topology and Concurrency*. Springer International Publishing, 2016. ISBN: 978-3-319-15397-1. DOI: 10.1007/978-3-319-15398-8. URL: http://www.springer.com/fr/book/9783319153971
- Samuel Mimram and Aly-Bora Ulusoy. *Sparkling*. Available at https://smimram.github.io/sparkling/. 2021

## Representations of the state space

- **Intervals** : $\mathcal{I}(X)$ = set of intervals $(s,t) = \{x \in X \mid s \leq x \leq t\}$ of $X$. Intervals are called finitely complemented when :

  $\exists S, T$ finite, such that $\{x \mid s \not\leq x\} = \downarrow S$ and $\{x \mid x \not\leq t\} = \uparrow T$
- **Regions** The regions $\mathcal{R}_{\mathcal{F}}(X)$, the finites sets of finitely complemented intervals of $X$, with order :

  $R \leq S \iff \forall r \in R, \exists s \in S, r \subseteq s$ **and** the converse implies $S \subseteq R$



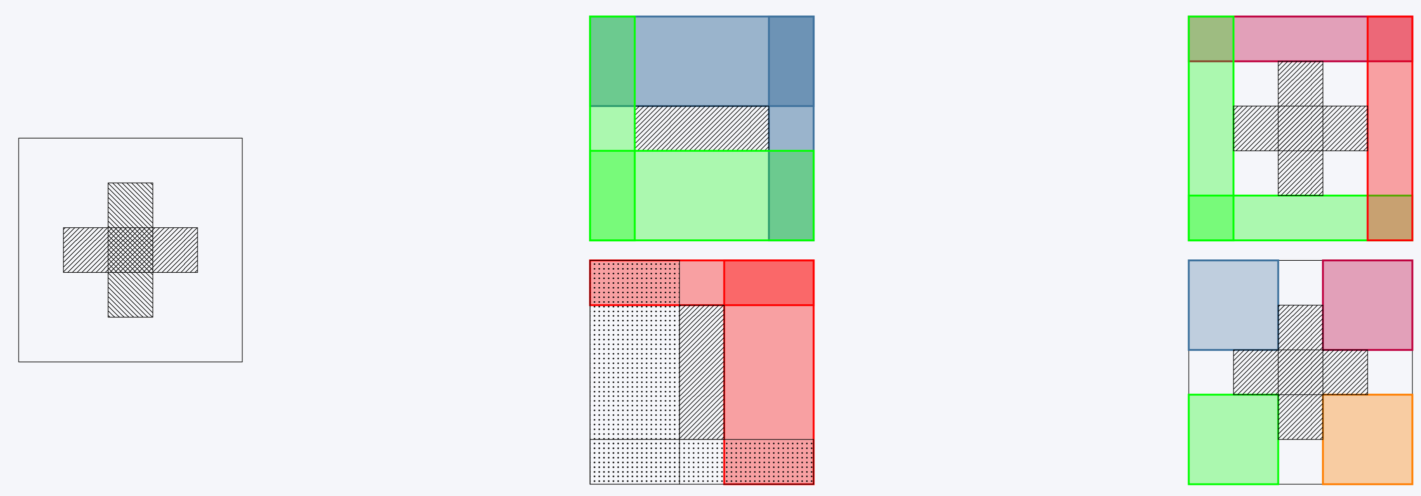
$R_1 = \{[0,\frac{1}{2}] \times [0,1], [\frac{1}{2},1] \times [0,1]\}$ $\quad R_2 = \{[0,1] \times [0,1]\}$ $\quad R_3 = R_2 \cup \{[\frac{1}{4},\frac{3}{4}] \times [\frac{1}{4},\frac{3}{4}]\}$

Figure 3. Example of regions covering $[0,1] \times [0,1]$. $R_2 \geq R_1$ and $R_2 \geq R_3$

There exists a "best" representative for each region that corresponds to the smallest set of maximal intervals that covers it. Above it is $R_2$.

## Algorithm for the Normal Form [2]

**Input:** A region $R$ of finitely complemented intervals
**Output:** Normal form of the complement of $[R]$
**for** *each interval* $(s,t)$ *of* $R$ **do**
  $I[(s,t)] = \{(\bot, z) \mid z \in \max\{x \mid x \not\geq s\}\} \cup \{(y, \top) \mid y \in \min\{x \mid x \not\leq t\}\}$
**end**
**for** *each multiset* $m \in \times_{r \in R} |I[r]|$ **do**
  Add $\bigcap_{r \in R} I[r][m[r]]$ to **result** if it is not contained in another interval of **result**
**end**
**return** *result*

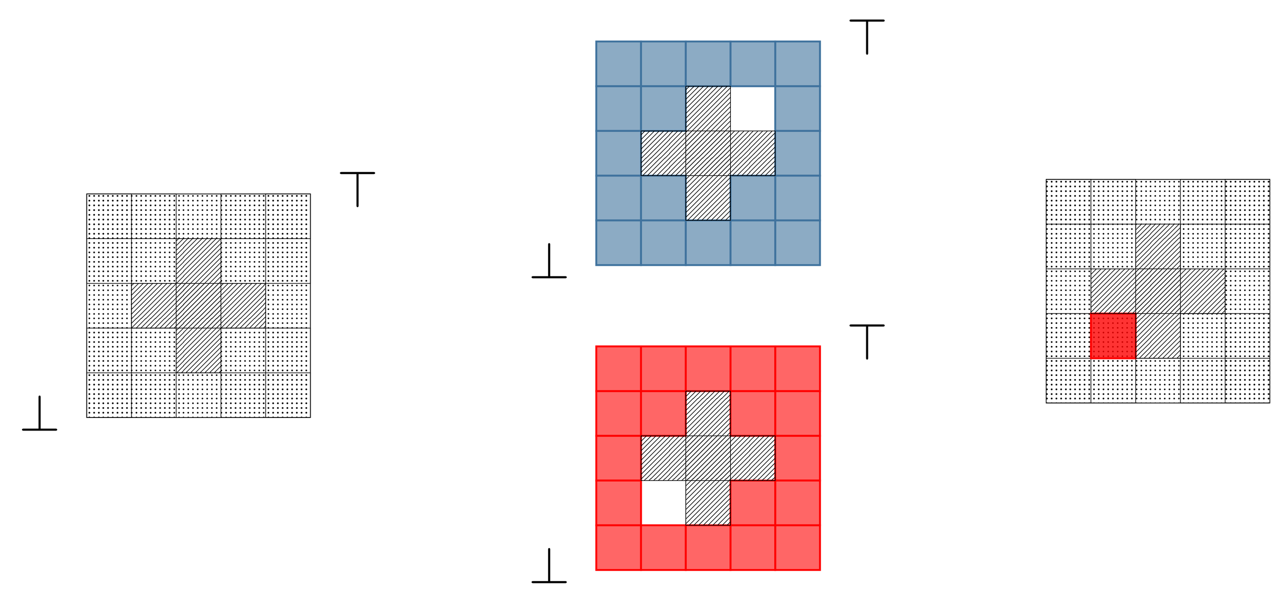


1. Cover of Forbidden 2. Compute Complements 3. Maximal Intersections

Figure 4. Algorithm applied to the swiss cross

## Deadlock Computation Algorithm [1]

1. Generate the finest partition of the normal form.
2. Any interval of the form $(\bot, x)$ is reachable and any other interval is **Reachable** if one of its position has a direct predecessor in **Reachable**.
3. **Deadlocks/Doomed** are **Reachable** intervals,not **Reachable** in the dual.




1. Normal Form 2. Reachability and Dual 3. Deadlocks!

Figure 5. Deadlock Computation on the Swiss Cross